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INTERNAL WAVES GENERATED BY LOCAL DISTURBANCES IN A LINEARLY STRATIFIED LIQUID OF FINITE DEPTH

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§1. In order to investigate the internal waves caused by the elongation of an axially symmetric body moving horizontally at constant velocity U in a stratified liquid, we consider the stationary problem of the flow of a uniform stream of heavy liquid of finite depth past a point source and sink of equal magnitude m which are situated below the free surface. The method of solving this problem is analogous to [1], in which we investigated the case of an unbounded liquid.

The source and sink are situated at a depth h below the unperturbed free surface, $y=0$, of the horizontal layer of liquid, $-\infty < x, z < \infty$, $-H \leq y \leq 0$. The line segment connecting the singularities is of length $2a$ and parallel to the x axis, which coincides with the direction of the velocity vector of the liquid far upstream. In the unperturbed state the distribution of the liquid density has the form

$$\rho_0(y) = \rho_s(1 - \alpha y), \quad -H \leqslant y \leqslant 0, \quad \alpha = \text{const} > 0. \tag{1.1}$$

We assume that for sufficiently deep immersion and weak stratification, the flow past this combination of source and sink is equivalent to the flow past a closed axially symmetric body (analogous to an unbounded homogeneous liquid). The radius R of the midsection, the elongation d of the body, and the velocity U of the fundamental stream uniquely determine the values of a and m [1].

In the linear formulation, making use of the Boussinesq approximation, the equations of motion have the form

$$\begin{gathered}\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = m[\delta(x+a) - \delta(x-a)]\delta(y+h)\delta(z), \\ \rho_s U\partial u/\partial x = -\partial p/\partial x, \quad \rho_s U\partial v/\partial x = -\partial p/\partial y - g\rho, \quad \rho_s U\partial w/\partial x = -\partial p\partial z, \\ U\partial\rho/\partial x - \alpha\rho_s v = 0\end{gathered} \tag{1.2}$$

with the boundary conditions

$$v = 0, \quad y = 0, \quad y = -H, \quad u, \ v, \ w, \ p, \ \rho \to 0, \quad x^2 + z^2 \to \infty,$$

where u, v, w, p, and ρ are the perturbations of the components of the velocity vector in the directions of the x, y, and z axes, the pressure, and the density which are caused by the presence of the singularities in the originally unperturbed flow; g is the acceleration of gravity; and δ is the Dirac delta function.

The free surface is replaced by a rigid "lid", since for sufficiently deep immersion the surface waves are negligibly small and the internal waves, for weak stratification, cause practically no distortion in the shape of the free surface [1, 2].

The function $\eta(x, y, z)$, determining the vertical deviation of a liquid particle from its unperturbed state, satisfies the linearized condition $\partial\eta/\partial x = v/U$.

Introducing the dimensionless variables $(x_*, y_*, z_*, h_*, H_*, \eta_*, a_*) = (1/R)(x, y, z, h, H, \eta, a)$, $(u_*, v_*, w_*) = (1/U)(u, v, w)$, and $m_* = m/UR^2$, we reduce Eq. (1.2) to a single equation for the function v_* (the subscript asterisk will be omitted from now on):

$$\frac{\partial^2}{\partial x^2}\Delta v + S\Delta_2 v = m\frac{\partial}{\partial y}\delta(y+h)\frac{\partial^2}{\partial x^2}[\delta(x+a) - \delta(x-a)]\delta(z),$$

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where $S = \alpha g R^2/U^2$; Δ and Δ_2 are, respectively, the three-dimensional and two-dimensional (with respect to x and z) Laplace operators.

Applying the Fourier transform

$$f(\mu, y, \nu) = \int_{-\infty}^{\infty} e^{-i\mu x} dx \int_{-\infty}^{\infty} e^{-i\nu z} v(x, y, z)\, dz \tag{1.3}$$

for real μ and ν, we obtain for the function f the ordinary differential equation

$$f'' - (k^2 - \lambda) f = 2im \sin \mu a \cdot \delta'(y + h)$$

with the boundary conditions

$$f = 0,\ y = 0,\ y = -H,$$

the solution of which has the form

$$f = im \sin \mu a \left[\operatorname{sgn}(y + h) e^{M|y+h|} + e^{-M(y+h)} - \frac{2\operatorname{sh} M(H + y) \cdot \operatorname{ch} Mh}{\operatorname{sh} MH} \right],$$

where $k^2 = \nu^2 + \mu^2$; $\lambda = Sk^2/\mu^2$; $M = (k^2 - \lambda)^{1/2}$.

Applying the inverse Fourier transform, for the function v(x, y, z) we obtain (in a manner analogous to [1])

$$v(x, y, z) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} e^{i\mu x} d\mu \int_{-\infty}^{\infty} e^{i\nu z} f d\nu = \frac{1}{\pi^2} \operatorname{Re} \int_0^{\infty} e^{i\mu x} d\mu \int_0^{\infty} \cos \nu z \cdot f\, d\nu, \tag{1.4}$$

or, introducing a change of variables,

$$\mu = k \sin \theta,\ \nu = k \cos \theta,\ x = r \cos \varphi,\ z = r \sin \varphi,$$

$$v(r, \varphi, y) = \frac{1}{2\pi^2} \operatorname{Re} \int_0^{\pi/2} d\theta \int_0^{\infty} k f(k, \theta) \left[e^{ikr \sin(\theta + \varphi)} + e^{ikr \sin(\theta - \varphi)} \right] dk. \tag{1.5}$$

In this problem, what is of chief interest is the investigation of internal waves at long distances behind the body; therefore, in (1.5), when we carry out the integration in the complex k-plane, we shall have left only simple integrals, residues of the integrand at the poles, which are the roots of the equation $\sinh MH = 0$. For $k > \lambda$ this equation has no real roots; for $0 \le k \le \lambda$ there are $N(\theta) = E[H\sqrt{S}/\pi \sin \theta]$ (E [] is the integral part of a number) positive roots $k_n = (S/\sin^2\theta - n^2\pi^2/H^2)^{1/2}$ $(n = 0, \ldots, N)$. Consequently,

$$v(r, \varphi, y) = A \left[\int_0^{\pi/2} \sum_{n=1}^{N} \sin(ak_n \sin \theta) \cos\left(\frac{n\pi h}{H}\right) \cdot BD_+ d\theta + \int_{\varphi}^{\pi/2} \sum_{n=1}^{N} \sin(ak_n \sin \theta) \cos\left(\frac{n\pi h}{H}\right) \cdot BD_- d\theta \right], \tag{1.6}$$

where $A = -2m/H^2$; $B = n \sin(n\pi y/H)$; $D_\pm = \cos(rk_n \sin(\theta \pm \varphi))$. The other desired functions u, w, and η will have analogous forms, and for u we will have

$$A = 2\pi m/H^3,\ B = n^2 \sin \theta \cos(n\pi y/H)/k_n,\ D_\pm = \sin(rk_n \sin(\theta \pm \varphi)),$$

for w we will have

$$A = 2\pi m/H^3,\ B = n^2 \cos \theta \cos(n\pi y/H)/k_n,\ D_\pm = \pm \sin(rk_n \sin(\theta \pm \varphi)),$$

and for η we will have

$$A = -2m/H^2,\ B = n \sin(n\pi y/H)/k_n \sin \theta,\ D_\pm = \sin(rk_n \sin(\theta \pm \varphi)).$$

In the case of an unbounded liquid we obtain, in particular, for $\eta(r, \varphi, \bar{y})$ $(\bar{y} = y + h)$

$$\eta(r, \varphi, \bar{y}) = -\frac{m}{\pi^2} \left[\int_0^{\pi/2} d\theta \int_0^{\sqrt{S}/\sin\theta} \Phi(k, \theta) \sin(rk \sin(\theta + \varphi))\, dk \pm \int_{\varphi}^{\pi/2} d\theta \int_0^{\sqrt{S}/\sin\theta} \Phi(k, \theta) \sin(rk \sin(\theta - \varphi))\, dk \right], \tag{1.7}$$

where $\Phi(k, \theta) = \sin(ak\sin\theta)\sin(\bar{y}\sqrt{S/\sin^2\theta - k^2})/\sin\theta$, which is analogous to formula (29) of [1] for $a\sqrt{S} \ll 1$. Applying the stationary-phase method for the asymptotic estimation of the integrals in (1.7) as $x^2 + y^2 + z^2 \to \infty$, we obtain

$$\eta(x, y, z) = \frac{m}{\pi}\sin\left(\frac{a\sqrt{S}xy}{(y^2+z^2)^{1/2}(x^2+y^2+z^2)^{1/2}}\right)\times$$
$$\times\frac{[x^2z^2+(y^2+z^2)^2]^{1/2}}{(y^2+z^2)(x^2+y^2+z^2)^{1/2}}\cos\left(y\sqrt{\frac{S(x^2+y^2+z^2)}{y^2+z^2}}\right), \tag{1.8}$$

which in the case of a dipole ($a=0$, $ma=\pi$) coincides with formula (6.10) of [3]. In [1] we gave the results of the numerical integration of (1.7); comparison of these with (1.8) shows that the asymptotic solution can be taken for $x \geqslant 4\pi/\sqrt{S}$.

The investigation of the integrals in (1.6) as $r \to \infty$ will be carried out by the stationary-phase method. To do this, we find the roots of the equation $\Psi'(\theta)=0$, where $\Psi(\theta)=k_n\sin(\theta\pm\varphi)$. For our determination of the stationary points, we obtain

$$\operatorname{tg}\varphi = \mp f(\theta), f(\theta) = \frac{b\cos\theta\sin^3\theta}{1-b\sin^4\theta}, b = \frac{n^2\pi^2}{SH^2}. \tag{1.9}$$

For $0 < \theta < \pi/2$ the function $f(\theta)$ is positive, and, consequently, stationary points are possible only for the second integral in (1.6). For $n \le n_0$ $[n_0 = N(\pi/2)]$ there are two stationary points $\theta_{1,2}$ if $\tan\varphi < f(\theta_m)$ $(\theta_1 < \theta_m < \theta_2)$ and one stationary point if $\tan\varphi = f(\theta_m)$; if $\tan\varphi > f(\theta_m)$, there are no stationary points $(\theta_m = \arcsin((2-\sqrt{4-3b})/b)^{1/2})$. For $n > n_0$ there exists one stationary point θ_1 if $\sin\varphi \leqslant 1/\sqrt{b}$.

The final expression for the function $\eta(r, \varphi, y)$ has the form

$$\eta(r, \varphi, y) = -\frac{2m}{H}\sqrt{\frac{2}{\pi r\cos\varphi}}\times\left[\sum_{n=1}^{n_0}A_n\sum_{i=1}^{2}B(k_{in}, \theta_{in})\times\right.$$
$$\left.\times\sin(rk_{in}\sin(\theta_{in}-\varphi)+(-1)^i\pi/4)+\sum_{n=n_0+1}^{N_1}A_nB(k_{1n}, \theta_{1n})\sin(rk_{1n}\sin(\theta_{1n}-\varphi)-\pi/4)\right], \tag{1.10}$$

where

$$N_1 = N(\varphi);\ A_n = \sin(n\pi y/H)\cos(n\pi h/H);$$
$$B(k, \theta) = \sin(ak\sin\theta)\cdot\left[\frac{1-b\sin^4\theta}{k\sin\theta\cdot|3-4\sin^2\theta+b\sin^4\theta|}\right]^{1/2}.$$

The expressions for the components of the velocity vector of the perturbed flow are analogous in form. It is interesting to note that for a liquid of finite depth the internal waves are concentrated within the angle $|\varphi| \le \varphi_m$, where φ_m is the maximum of the values $\arcsin(H\sqrt{S}/\pi(n_0+1))$ and $\operatorname{arc\,tan} f(\theta_m(n_0))$, while in the case of an unbounded liquid they cover the entire half-space $x > 0$.

The isocurves of the function $5\eta/\sqrt{S}R$ for the value $x\sqrt{S}/R = 15$ are shown in Fig. 1 for $H\sqrt{S}/R = 3$, $h\sqrt{S}/R = 1$, $d = 1$. The roots of Eq. (1.9) were determined numerically. Unlike the case of an unbounded liquid, the wave wake becomes asymmetric for $h \ne H/2$, broader, and more subdivided.

§2. To investigate the internal waves induced by the collapse of the mixing zone, we consider a plane nonstationary problem of flow arising as a result of the collapse of an initially circular spot of completely or partially mixed liquid surrounded by a liquid which is linearly stratified in density. A number of studies have been published on this problem. The authors of [4-6] give the results of the numerical solution of a complete system of Navier–Stokes equations (in [4, 6] the Boussinesq approximation is used), and the authors of [3, 7-10] make an analytical investigation of the collapse process on the basis of the linearized equations of a nonviscous liquid ([3, 8, 9] consider an unbounded liquid, while [7, 10] deal with a liquid layer of finite depth). In [10], unlike [3, 7-9], the Boussinesq approximation is not used, and a numerical comparison with [7] shows that the difference between these solutions is insignificant.

In the present study the internal waves generated by the collapse have been investigated by a method analogous to that of Sec. 1, and the results obtained are more suitable for numerical integration than those of [7].

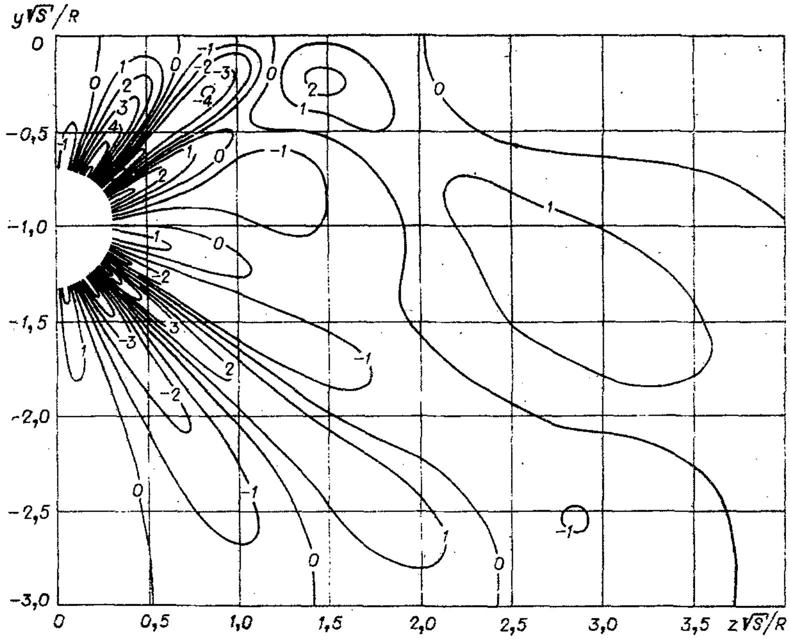

Fig. 1

In the plane of the motion we set up a Cartesian system of coordinates y, z (the y axis is directed vertically upward, and the z axis coincides with the free surface). At the initial instant of time $t=0$, within a circle of radius R with center at the point $y=-h$, $z=0$ $(h>R)$, the liquid is mixed, so that the density perturbation has the form

$$\rho(y, z, 0)=\begin{cases}\varepsilon\alpha(y+h)\rho_s, & \sqrt{(y+h)^2+z^2}<R,\\ 0, & \sqrt{(y+h)^2+z^2}>R, \quad 0<\varepsilon\leqslant 1,\end{cases}$$

where in the case of a completely mixed spot $\varepsilon=1$, and outside the spot the distribution of density is determined by (1.1).

The linearized equations of motion in the Boussinesq approximation have the form

$$\begin{gathered}\partial v/\partial y+\partial w/\partial z=0,\ \rho_s\partial v/\partial t=-\partial p/\partial y-g\rho,\\ \rho_s\partial w/\partial t=-\partial p/\partial z,\ \partial\rho/\partial t-\alpha\rho_s v=0\end{gathered}\tag{2.1}$$

with the boundary conditions

$$v=0,\ y=0,\ y=-H,\ v,\ w,\ \rho,\ p\to 0,\ \ z\to\pm\infty$$

and the initial conditions

$$v=w=p=0,\ \rho=\rho_0(y)F(y,\ z),\ t=0,$$

where

$$\begin{gathered}F(y,\ z)=\varepsilon[\rho_0(-h)/\rho_0(y)-1]H(y+h+R)H(R-y-h)H(z+\\ +\sqrt{R^2-(y+h)^2})H(\sqrt{R^2-(y+h)^2}-z).\end{gathered}$$

Here, as before, v, w, p, and ρ are the perturbations of the components of the velocity vector in the directions of the y and x axes, the pressure, and the density; H() is the Heaviside step function.

The vertical deviation of a liquid particle from its unperturbed state, $\eta(y, z, t)$, is determined from the linearized condition $\partial\eta/\partial t=v$.

Introducing the dimensionless variables

$$(y_*,\ z_*,\ h_*,\ H_*,\ \eta_*)=(1/R)(y,\ z,\ h,\ H,\ \eta),\ t_*=\sqrt{g\alpha}t,\ (v_*,\ w_*)=(1/R\sqrt{g\alpha})\ (v,\ w)$$

(from this point on we shall omit the asterisk), reducing the equations (2.1) to a single equation for v, and applying, as in Sec. 1, the double Fourier transform (1.3) (the time t plays the role of the longitudinal coordinate x in the preceding discussions), we obtain

$$f'' - \nu^2\left(1 - \frac{1}{\mu^2}\right)f = -\frac{2\varepsilon\nu}{\mu^2}(y+h)H(y+h+1)H(1-y-h)\sin\left(\nu\sqrt{1-(y+h)^2}\right) \tag{2.2}$$

with boundary conditions

$$f = 0,\ y = 0,\ y = -H.$$

The solution of Eq. (2.2) outside the spot has the form

$$f = \pi\varepsilon J_2(\nu/\mu)[\mathrm{sgn}(y+h)e^{M|y+h|} + e^{-M(y+h)} - 2\mathrm{ch}Mh \cdot \mathrm{sh}M(y+H)/\mathrm{sh}MH],$$

where $M = \nu\sqrt{1-1/\mu^2}$; J_2 is the Bessel function of the first kind and second order.

In the problem, as before, what we are mainly interested in is the investigation of the internal waves far from the perturbation, and therefore in carrying out the inverse Fourier transforms, analogous to (1.4) and (1.5), we have left only simple integrals, which are residues at the poles of the integrand. The desired solution has the form

$$v(r, \varphi, y) = A\left[\int_0^{\pi/2} J_2(\mathrm{ctg}\,\theta)\sum_{n=1}^{N}\cos\left(\frac{n\pi h}{H}\right)\cdot BD_+\,d\theta + \int_{\varphi}^{\pi/2} J_2(\mathrm{ctg}\,\theta)\sum_{n=1}^{N}\cos\left(\frac{n\pi h}{H}\right)\cdot BD_-\,d\theta\right],$$

where

$$A = -\frac{2\pi\varepsilon}{H^2};\ B = \frac{n}{\cos^2\theta}\sin\left(\frac{n\pi y}{H}\right);\ D_\pm = \sin(rk_n \sin(\theta \pm \varphi));$$

$$N = E[H\,\mathrm{ctg}\,\theta/\pi];\ k_n = \sqrt{1-(n\pi\,\mathrm{tg}\,\theta/H)^2}/\sin\theta.$$

The other desired functions have analogous forms; for w

$$A = -\frac{2\pi^2\varepsilon}{H^3},\ B = \frac{n^2}{k_n\cos^3\theta}\cos\left(\frac{n\pi y}{H}\right),\ D_\pm = \pm\cos(rk_n\sin(\theta\pm\varphi)),$$

for η

$$A = \frac{2\pi\varepsilon}{H^2},\quad B = \frac{n}{k_n \sin\theta\cos^2\theta}\sin\left(\frac{n\pi y}{H}\right),\ D_\pm = \cos(rk_n\sin(\theta+\varphi)).$$

In the case of an unbounded liquid we find, in particular, for

$$\eta(r, \varphi, \bar{y})\ (\bar{y} = y + h)$$

$$\eta(r, \varphi, \bar{y}) = \frac{\varepsilon}{\pi}\left[\int_0^{\pi/2} d\theta \int_0^{1/\sin\theta} \Phi(k, \theta)\cos(rk\sin(\theta+\varphi))\,dk + \right.$$
$$\left. + \int_{\varphi}^{\pi/2} d\theta\int_0^{1/\sin\theta} \Phi(k, \theta)\cos(rk\sin(\theta-\varphi))\,dk\right],$$

where $\Phi(k, \theta) = J_2(\mathrm{ctg}\,\theta)\sin\left(\bar{y}\cos\theta\sqrt{1/\sin^2\theta - k^2}\right)/\sin\theta$. An asymptotic estimate of these integrals by the stationary method for $y^2+z^2+t^2 \to \infty$ yields

$$\eta = \frac{\varepsilon z}{y^2+z^2}J_2\left(\frac{zt}{y^2+z^2}\right)\sin\left(\frac{yt}{\sqrt{y^2+z^2}}\right). \tag{2.4}$$

This expression coincides, to within a factor of 2, with formula (26) of [8]. In [3], in the investigation of the internal waves generated by the collapse of a completely mixed spot of radius R, the latter was limited by a quadrupole with moment $\pi R^4/4$. It was shown that at $t\to\infty$,

$$\eta = \frac{z^3t^2}{8(y^2+z^2)^3}\sin\left(\frac{yt}{\sqrt{y^2+z^2}}\right),$$

which coincide with (2.4) for $zt \ll y^2+z^2$.

Equation (2.3) was integrated numerically (considerable difficulties were encountered in using the stationary-phase method for an asymptotic estimate of the integrals), and the isocurves of the function $50\eta/R$ for the value $t\sqrt{\alpha g} = 15$ are shown in Fig. 2 for $H/R = 10$, $h/R = 4$, and $\varepsilon = 1$.

The shaded half-disk corresponds to the original position of the spot. Qualitatively, Fig. 2 is a repetition of Fig. 1. In Figs. 1 and 2, in view of the difficulty of the graphical mapping, we do not show the flow in the neighborhood of the point $y = -h$, $z = 0$. It should be noted that the flow in this region is described with sufficient accuracy by the solutions (1.8) and (2.4), respectively.

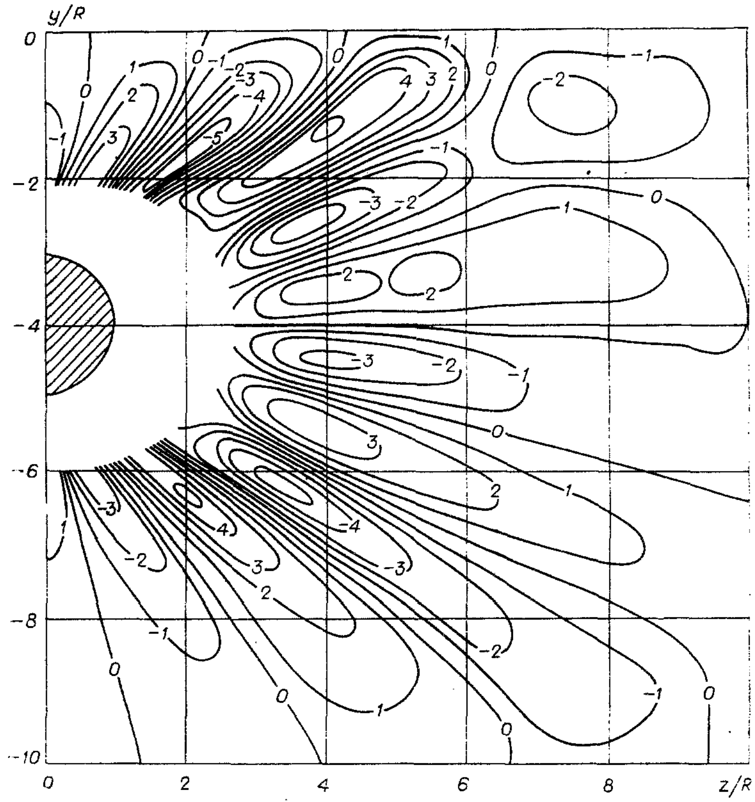

Fig. 2

We can also propose another method for solving these problems which is based on the use of the method of reflections. In investigating the internal waves far from the source of perturbation, knowing the solutions (1.8) and (2.4) and introducing in an unbounded liquid an infinite series of imaginary perturbations, equivalent to the initial perturbation and symmetrically arranged with respect to the horizontal planes $y = 0$ and $y = -H$, we obtain for the case of a liquid of finite depth a solution in the form

$$\eta(x, y, z) = \bar{\eta}(x, y, z) + \sum_{n=1}^{\infty} \left[\bar{\eta}(x, y + 2nH, z) + \bar{\eta}(x, y - 2nH, z)\right], \tag{2.5}$$

where $\bar{\eta}(x, y, z) = \eta_0(x, y + h, z) + \eta_0(x, y - h, z)$, $\eta_0(x, y, z)$ is the solution (1.8) for the first type of perturbations or (2.4) for the second type of perturbations [in this case, in (2.5) the variable x must be replaced by t]. A comparison of the results obtained by using (2.5) and the solutions (1.10), (2.3) yields good agreement.

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INTERACTION BETWEEN TURBULENT BOUNDARY LAYERS IN A RIGHT DIHEDRAL CORNER

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UDC 532.526.4

The interaction between two adjacent turbulent boundary layers which occurs in longitudinal flow over intersecting surfaces pertains to complex forms of viscous flow. Such flow is very often encountered in practice, for instance, at the joints between individual parts of aircraft, in flow around cross-shaped and V-shaped wings, etc. However, in spite of its practical importance, the nature of viscous interaction in corner configurations has not yet been investigated experimentally to a sufficient extent. As a rule, no allowance is made for the three-dimensional nature of corner flow in theoretical investigations, and, therefore, the results are in poor agreement with experimental data.

The present article is concerned with an experimental investigation of the integral characteristics of the boundary layer, determination of the extent of the interaction zone for different Reynolds numbers, and a study of the effect of the longitudinal pressure gradient.

The experiments are performed in the low-turbulence T-324 aerodynamic tunnel at the Institute of Theoretical and Applied Mechanics, Siberian Branch, Academy of Sciences of the USSR [1], using a right dihedral simulator (Fig. 1). Take-off openings, 3, with a diameter of 0.5 mm are provided on both sides 1 for measuring the static pressure. Both the fore and aft parts of the dihedral sides have a semielliptical shape with a 1 : 12 ratio of the semiaxes. The static pressure along the length of the simulator is varied by means of two rear-end flaps, 2. A clear-plastic dummy wall, 4, is mounted in the working section of the aerodynamic tunnel in order to ensure the assigned longitudinal pressure gradient at the surface of the simulator. The pressure gradient varies according to the degree to which the operating section is blocked, while the gradient sign is determined by the shape of the contour of the operating section artifically created by the dummy wall. Both positive and negative static pressure gradients $d\bar{p}/dx$ ($\bar{p} = (p - p_\infty)/q_\infty$ is the pressure coefficient) can thereby be created at the simulator surface.

The experiments are performed at unperturbed flow velocities from 10 to 52 m/sec, which corresponds to individual Reynolds numbers $Re_1 = (0.7-3.2) \cdot 10^6$ m^{-1}. A well-developed turbulent boundary layer is produced by means of a turbulence generator consisting of coarse-grained emery paper 10 mm wide, which is pasted on along the spread of the corner at a distance of 10 mm from the leading edge.

The total and the static pressures and the direction of the velocity vector in the boundary layer are measured by means of miniature pneumatic tubes, 5, the geometric characteristics of which are shown in Fig. 1. Special calibration checks have shown that, with an accuracy to 1%, the flat and the cylindrical tubes are not sensitive to downwashes to up to 9° and 22°, respectively. Similar calibrations have also been performed in the investigated velocity range for a double-barrelled pneumatic tube, which is used for determining the direction of the velocity vector in the boundary layer of the dihedral corner. The thus obtained data on the downwash angles in two mutually perpendicular planes and the knowledge of the longitudinal velocity component make it possible to determine the transverse velocity component.

In order to verify the hypothesis concerning the constancy of static pressure across the boundary layer, we measured the static pressure profiles by means of a special microtube, which was also calibrated beforehand. The results of these experiments have shown that the maximum change in static pressure along the height of the boundary layer occurs in the bisecting plane and is equal to $\pm 0.007 q_\infty$. Allowance for this degree

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